

NON-BPS BLACK HOLES AND SELF-INTERACTING FUNDAMENTAL STRINGS

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The string-black hole correspondence principle can be investigated in the non-BPS scenario by studying the string configuration and entropy when the string coupling is slowly increased. Through a rigorous analysis, it is shown how an ensemble of string states at fixed mass and Neveu-Schwarz charges gets dominated in any dimension by compact states for which the one-loop corrections are important (possibly signaling the transition to a black hole regime/description) and with a size (spread) within the horizon radius of the expected correspondent black holes.

The relation between the Hawking-Bekenstein entropy of black holes and the counting of microstates is one of the most studied topics in black holes physics. In this short communication we deal with the non-BPS scenario in the setup of closed superstring theory. In doing so, we touch upon some important questions: how to individuate the relevant microstates, how and when the two descriptions (general relativity and quantum string) match and, finally, how geometric features of black holes arise from the quantum description.

Within string theory, the string-black hole correspondence principle¹ states that a classical black hole is in one-to-one correspondence with an ensemble of string and/or D-brane states (depending on its type of charges).

Two quantities in particular play a key role in the analysis: the string coupling g_s ^a and the black hole horizon radius R_{BH} , especially in relation with the string length scale^b $l_s = \sqrt{\alpha'}$.

The string and the black hole entropy are conjectured to become equal at a correspondence point $g_s \sim (M^2 - Q^2)^{-\frac{1}{4}} \equiv g_c$ in parameter space, where M is the mass of the string states in the ensemble, equal to the black hole's one at this point, and Q^i are the Neveu-Schwarz charges^c. Also, at this point R_{BH} approaches l_s .

There are major obstacles to verifying such a statement within string theory. We must show what happens to the relevant ensemble of string states and their entropy^d when the string coupling (adiabatically) increases^e from $g_s = 0$ to $g_s = g_c$.

^aLinked to the Newton's constant and the string length at the perturbative level as $G_N \sim g_s^2(\alpha')^{\frac{d-1}{2}}$ in $d = D - 1$ extended spatial dimensions.

^bIn what follows the horizon radius is to be intended in the *string frame*. The general relativity description is valid when the curvature at R_{BH} (in the string frame) is smaller than the string scale, this yields the condition $R_{BH} > l_s$.

^cHere, for simplicity $|Q_L^i| = |Q_R^i| = |Q^i|$, $Q^2 = \sum (Q^i)^2$, where $L(R)$ refer to the (anti)holomorphic part of the string.

^dWe will consider single-string entropy, which dominates the contribution to the total entropy.

^eThis is what makes the comparison safest in the BPS case, due to the non-renormalization properties of BPS state counting. In the non-BPS case, instead, one has to compute the corrections to the microstate counting when interactions are turned on.

This entails coping with two main issues: i) computing the relevant corrections to the tree-level results due to interactions, ii) correctly defining and measuring the size of string states and their *size distribution* and check its relation with the horizon radius of the expected correspondent black hole.

Refs. 2,3 investigated these issues within string theory. In fact, these problems were previously treated in approximated models, leading to a variety of results,⁴ but never in fully rigorous string formalism.

In coping with ii), the traditional approach of defining a size operator for the string failed to comply with the requirements of the theory. Ref. 2, instead, measured the size of an object through a well-defined operational procedure. The basic idea is that, in a theory where the observables are scattering amplitudes, the size of an object is measured by scattering probes^f over it. For mixed states (ensembles), the form factor obtained in this way can be related to their size distribution.

In a microcanonical ensemble^g, the entropy of the superstring at zero coupling and fixed (tree) mass level $M_0^2 = Q_{L(R)}^2 + N_{L(R)}$, Neveu-Schwarz charges $Q_{L(R)}^i$ and size R was found to be ($\alpha' = 4$, $\mathcal{N} = \sqrt{N_L} + \sqrt{N_R}$)

$$S = \pi \mathcal{N} \sqrt{d-1} - \frac{3\sqrt{d-1}}{8\mathcal{N}\pi} R^2 + \ln \left(\frac{R^{d-1}}{N_L^{\frac{d+2}{4}} N_R^{\frac{d+2}{4}} \mathcal{N}^{\frac{d}{2}}} \right) \quad (1)$$

Turning on interactions ($g_s \neq 0$) yields corrections to the free-string result. Neglecting those related to R (renormalization of the size), the relevant ones come from the string mass-shift.

The study of mass shifts in string theory is very difficult because very few physical vertex operators for massive string states are actually known (in covariant gauge) and also because of the intrinsic complications in the computations of amplitudes.⁵

In order to obtain the average squared mass shift for the relevant ensemble of states at fixed tree-level mass charges and size, Ref. 3 exploited two key properties of the string amplitudes. First, the unitarity of the string S-matrix and, then, the modular and periodicity properties of the string amplitudes on the torus, which allowed to determine a system of recurrent equations for the amplitude itself^h.

Eventually, the average squared mass shift for the ensemble of string states at fixed mass and chargesⁱ was found³ to be^j

$$\overline{\Delta M^2}_{|N,Q} = -g_s^2 (M^2 - Q^2)^{1+\frac{3-P}{4}}, \quad (2)$$

^fIn our case, a linear combination of graviton, dilaton and Kalb-Ramond field as we are interested in the mass distribution of the object.

^gWhich appears to be the only well-defined one for closed string states due to the level matching conditions.

^hNamely for its coefficients, once the amplitude is expanded in powers of suitable variables, see Ref. 3.

ⁱAgain, we consider for simplicity $|Q_L^i| = |Q_R^i| = |Q^i|$, $N_L = N_R = N$.

^jThis formula tells us that the interaction in the effective Lagrangian will become important – of order one – at $g_{se} \sim (M^2 - Q^2)^{\frac{d-6}{8}}$ as expected from field theory considerations. We are writing the result in terms of the physical mass M , which is probably an even more accurate estimate.

while the average squared mass shift for the ensemble of states at fixed M, Q^i, R is²

$$\overline{\Delta M^2}_{|M,Q,R} = -g_s^2(M^2 - Q^2)R^{2-d}, \quad (3)$$

absorbing an inessential proportionality constant in g_s . We can then infer the corrections to the single-string entropy S when the g_s is adiabatically increased²

$$e^S \sim e^{2\pi\sqrt{d-1}\left(\sqrt{M^2-Q^2} + g_s^2 \frac{\sqrt{M^2-Q^2}}{2R^{d-2}} - \frac{3}{32\pi^2\sqrt{M^2-Q^2}}R^2\right)} \quad (4)$$

In particular, we see that for $2^{-1}g_s^2\sqrt{M^2-Q^2}R^{2-d} \geq 1$, the entropy becomes dominated by strings whose size is $R \lesssim R_b \sim (g_s^2\sqrt{M^2-Q^2})^{\frac{1}{d-2}}$, which has the same perturbative formula of the horizon radius^k of the correspondent black hole. It approaches the string length at $g_s \sim (M^2 - Q^2)^{-\frac{1}{4}}$, in accordance with the correspondence principle.

The string entropy is dominated in particular by the states with the minimum possible size. It can be shown² that perturbation theory is sufficient to determine the minimal size only in $d = 3$, yielding $R_{\min} = \frac{2}{g_s^2(M^2-Q^2)}$. It is finally interesting to observe that the relative average one-loop mass shift for these minimum size states is of order $\frac{\overline{\Delta M^2}_{|M,Q,R_{\min}}}{M^2} \sim O(1)$, possibly signaling a radical change in the relation between entropy and mass for $g_s > g_c$ and hinting to a transition between a perturbative string regime and a black hole one. Finally, the corrected formula for the entropy at leading order in $d = 3$ is²

$$S(M, Q) \sim 2\pi\sqrt{2}\sqrt{M^2 - Q^2} \left(1 + \frac{g_s^4(M^2 - Q^2)}{4}\right). \quad (5)$$

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^kIn the Einstein frame, which must be used to compute the black hole entropy.